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QUESTION BANK (DESCRIPTIVE)

Subject with Code : Transform & Discrete Mathematics (18HS0832)

Course & Branch : B.Tech CE&AG

Year &Sem : II-B.Tech& I-Sem **Regulation : R18**

UNIT-I

TRANSFORM CALCULUS-I

1. a) Find the Laplace transform of $e^{at}\cosh bt$ [2 M]
   b).Find the Laplace transform of $3\cos 3t.\cos 4t$ [2M]
   c) Find $L^{-1}\left\{\frac{2s-5}{4s^2+25}\right\}$ by using linear property. [2 M]
   d) Find $L\{t^2+3t+10\}$ [2M]
   e)State Convolution theorem. [2M]
2. a) Find the Laplace transform of $e^{-3t}\left(2\cos 5t-3\sin 5t\right)$ [5M]
   b) Find the Laplace transform of $f(t)=\int_0^t e^{-t}\cos t\,dt$. [5M]
3. a) Find the Laplace transform of $f(t)=\frac{1-\cos at}{t}$ [5 M]
   b) Show that $\int_0^\infty t^2e^{-4t}.\sin 2t\,dt=\frac{11}{500}$, Using Laplace transform [5 M]
4. a)Find Laplace Transform of Square-wave function of period 2a,
   defined as $f(t)=\begin{cases} k, & 0<t<a \\ -k, & a<t<2a \end{cases}$ [5 M]
   b) Using Laplace transform, evaluate $\int_0^\infty \frac{\cos at-\cos bt}{t}dt$. [5 M]
5. a) Find the Laplace transform of $f(t)=e^{-4t}\int_0^t \frac{\sin 3t}{t}dt$. [5 M]
   b) Find the Laplace transform of $f(t)=t\ e^{2t}\sin 3t$ [5 M]
6. a) Find $L^{-1}\left\{\frac{3s-2}{s^2-4s+20}\right\}$ by using first shifting theorem [5M]
   b) Find $L^{-1}\left\{\frac{1}{2}\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\}$ [5M]
7. a) Find $L^{-1}\left\{\frac{1}{\left(s^2+5^2\right)^2}\right\}$, using Convolution theorem. [5 M]

b) Find $L^{-1}\left\{\frac{s^2}{(s^2+\ 4)\ (s^2+25)}\right\}$, using Convolution theorem. [5M]

8. a) Find the Inverse Laplace transform of $\frac{1}{s^2(s^2+a^2)}$. [5 M]

b) Find $L^{-1}\left\{\log\left(\frac{s-1}{s+1}\right)\right\}$ [5 M]

9. Use transform method to solve $(D^2+5D+6)y=5e^t$ where $y(0)=2, y^1(0)=1$ [10M]

10. Solve the D.E $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=5\sin t$ using Laplace Transform given that $y(0)=0, y^1(0)=0$ [10M]

UNIT II

Fourier Transforms

1. a) Find the Fourier sine transform of $e^{-|x|}$ Hence show that $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}, m > 0.$
   b) Find the Fourier cosine transform of $2e^{-5x} + 5e^{-2x}$.
2. Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 |x| < a \\ 0 \quad |x| > a > 0 \end{cases}$
   Hence show that $\int_0^{\infty} \frac{\sin x - x\cos x}{x^3} dx = \frac{\pi}{4}$.
3. a) Find the Fourier transform of $f(x) = e^{\frac{-x^2}{2}}, -\infty < x < \infty$
   b) If $F(p)$ is the complex Fourier transform of $f(x)$, then prove that the complex fourier transform of $f(x) \cos xa$ is $\frac{1}{2}[F_s(p+a) + F_s(P-a)]$.
4. a) Find the Fourier cosine transform of $e^{-ax} \cos ax, a > 0$.
   b) Find the Fourier cosine transform of $f(x) = \begin{cases} x & for\ 0 < x < 1 \\ 2 - x & for\ 1 < x < 2. \\ 0 & for\ x > 2 \end{cases}$
5. Find the Fourier sine and cosine transforms of $f(x) = \frac{e^{-ax}}{x}$ and deduce that
$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} \sin sx\, dx = \tan^{-1}\left(\frac{s}{a}\right) - \tan^{-1}\left(\frac{s}{b}\right).$$
6. Find the Fourier sine and cosine transforms of $f(x) = e^{-ax}, a > 0$
   and hence deduce the integrals $(i) \int_0^{\infty} \frac{p \sin px}{a^2+p^2} dp$ $(ii) \int_0^{\infty} \frac{\cos px}{a^2+p^2} dp$
7. Find the Inverse Fourier sine transform of $f(x)$ of $F_s(p) = \frac{p}{1+p^2}$.
8. a) Find the finite Fourier sine transform of $f(x) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$.
   b) Find the inverse finite Fourier sine transform of $f(x)$, if $F_s(n) = \frac{16(-1)^{n-1}}{n^3}$, where n is a positive integer and $0 < x < 8$.
9. a)Find the Finite cosine transform of $f(x) = e^{ax} in\ (0, l)$.
   b) Find the Inverse Fourier cosine transform $f(x)$ if $F_c(n) = \frac{\cos\left(\frac{2n\pi}{3}\right)}{(2n+1)^2}$ where $0 < x < 4$.
10. a) Write the formula for Inverse Fourier Transform.
    b) Show that $F_s\{xf(x)\} = -\frac{d}{ds}\{F_c(s)\}$.
    c) Write the formula for Finite Fourier cosine transform.
    d) Find the Inverse Fourier sine transform f(x) if $F_s(n) = \frac{1-\cos n\pi}{n^2\pi^2}$ $0 < x < \pi$.
    e) Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 0 & -\infty < x < \alpha \\ x & \alpha < x < \beta \\ 0 & x > \beta \end{cases}$

UNIT-III

ALGEBRAIC STRUCTURES

1. Define group and an abelian group. Prove that the set Z of all integers with the binary operation $*$, defined as $a*b = a+b+1, \forall a,b \in Z$ is an abelian group.
2. a) Define and give an examples for group, semigroup, subgroup &abelian group.

   b)Let s={a,b,c} and let *denotes a binary operation on 's' is given below also let p={1,2,3}and addition be a binary operation on 'p' is given below.show that (s,*) & (p,(+)) are isomorphism

*	A	B	C
A	A	B	C
B	B	B	C
C	C	B	C

(+)	1	2	3
1	1	2	1
2	1	2	2
3	1	2	3

3. a)Show that the set={1,2,3,4,5} is not a group under addition & multiplication modulo 6

   b) On the set Q of all rational number operation * is defined by a*b=a+b-ab.

   Show that this operation Q forms a commutative monoid.

4. a) Explain the concepts of homomorphism and isomorphism of groups with examples.

   b) Let$(G,*)$ and (H,Δ) be a groups and $g: G \to H$ be a homomorphism. The the kernerl of g is a normal subgroup.

5 a)The necessary and sufficient condition for a non-empty sub-set H of a Group (G,*) to be a sub group is aϵH, bϵH=>a*b^{-1}ЄH

   b) Show that every homomorphic image of an abelian group is abelian.

6. a) Show that the set of all roots of the equation $x^4 = 1$ forms a group under multiplication.

   b) Show that the set of all rational numbers forms an abelian group under the composition defined by $a * b = \frac{ab}{2}$

7. a) In a group G for a,bϵG,o(a)=5, b≠e and aba^{-1}=b^2. Show that o(b)=31.

   b) Let $s = \{a, b\}$ be a set consider all possible permutations of S as $s_2 = \{P_1, P_2\}$ Show that $(s_2,*)$is a permutation group.

8. a) Let $Z_5^* = \{[1],[2],[3],[4]\}$ in which $[1],[2],\ldots..$ have the same meaning as in Z_5 except that $Z_5^* = Z_5 - \{[0]\}$. Also let X_5 is multiplication modulo 5. Show that $g: Z_4 \to Z_5^*$ is given by $g([0]) = [1], g([1]) = [2], g([2]) = [4], g([3]) = [3]$defines a homomorphism from the group $(Z_4, +_4) to (Z_5^*,*_4)$.Hence show that g is group isomorphic.

b) Show that if a, b are arbitrary elements of a group G then $(ab)^2=a^2b^2$ iff G is abelian.

9.a) Prove that the order of a subgroup of a finite group divides the order of the group ?

b) Prove that the kernel of a homomorphism from (G , *) to (H , Δ) is a subgroup of (G , *) .

10.(a) Define Monoid , Semi group?

(b) Let $G = \{1, -1, i, -i\}$ be a multiplicative group. Find the order of every element.

(c) Define isomorphism of a group?

(d)Define Normal group?

(e) Define Homomorphism of a semi group?

UNIT IV

Introduction to counting

1. (a)In how many ways can a committee of 5 teachers and 4 students be chosen from 9 teachers and 15 students with at least 2 students in each committee ?

   (b) How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where

   (i) each $x_i \geq 2$? (ii) Each xi> 2?

2. (a) How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8 and 9 if no Repetitions are allowed?

   (b)Find the generating function for the sequence 1,1,1,3,1,1,…………

3. (a) The question paper of mathematics contains two questions divided into two groups of 5 questions each. In how many ways can an examine answer six questions taking at least two questions from each group.

   (b) How many permutations can be formed out of the letters of word "SUNDAY"? How many of these (i) Begin with S? (ii) end with Y? (iii) Begin with S & end with Y? (iv) S &Y always together?

4. (a) In how many ways can the letters of the word COMPUTER be arranged? How many of them begin with C and end with R? how many of them do not begin with C but end with R?

   (b) Out of 9 girls and 15 boys how many different committees can be formed each consisting of 6 boys and 4 girls?

5. a) Determine the number of positive integer $1 \leq n \leq 100$ and is not divisible by 2 , 3 , or 5

   b) Solve $a_n = a_{n-1} + 2a_{n-2}, n \geq 2$ with initial conditions $a_0 = 0, a_1 = 1$

6. a)Solve $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$ with initial conditions $a_0 = 0, a_3 = 3, a_5 = 10$

   b) A survey among 100 students shows that of the three ice cream flavours vanilla, Chocolate, strawberry 50 students like vanilla, 43 like chocolate, 28 like strawberry, 13 like vanilla and chocolate, 11 like chocolate and straw berry, 12 like straw berry and vanilla and 5 like all of them.

   Find number of students who like each of the following flavours

   i) Chocolate but not straw berry ii) chocolate and straw berry but not vanilla

   iii) Vanilla or chocolate but not straw berry.

7. (a) Find how many integers between 1 and 60 that are divisible by 2 nor by 3 and nor by 5. Also determine the number of integers divisible by 5 not by 2, not by 3.

   b) Out of 80 students in a class, 60 play football, 53 play hockey and 35 both the games. how many students (i) do not play of these games. (ii) Play only hockey but not football.

8. a)Applying pigeon hole principle show that of any 14 integers are selected from the set $S = \{1,2,3,\dots\dots,25\}$ there are at least two whose seem is 26. Also write a statement that generalizes this result.

   b) Show that if 8 people are in a room, at least two of them have birthdays that occur on the same day of the week

9. a) Determine the sequence generated by

   $f(x) = 2e^x + 3x^2$ (ii) $7 e^{8x} – 4 e^{3x}$.

   b) Solve the RR $a_{n+2} - 2 a_{n+1} + a_n = 2^n$ with initial condition $a_0 = 2$ & $a_1 = 1$.

10. (a) State Pigeon?

    (b) State Multinomial theorem?

    (c) Define permutation & Combination?

    (d)State Generating Function?

    (e) State Inclusion and Exclusion?

UNIT-5
Introduction to Graphs

1. a) Define isomorphism. Explain Isomorphism of graphs with a suitable example.

b) Explain graph coloring and chromatic number give an example.

2. a)Explain about complete graph and planar graph with an example

b) Define the following graph with one suitable example for each graphs

(i) Spanning tree (ii) sub graph (iii) induced sub graph (iv) spanning sub graph

3. a) Explain in degree and out degree of graph. Also explain about the adjacency matrix representation of graphs. Illustrate with an example?

b) Give an example of a graph that has neither an Eulerian circuit nor a Hamiltonian circuit

4. a) Show that the maximum number of edges in a simple graph with n vertices is n (n-1) / 2

b) A graph G has 21 edges, 3 vertices of degree4 and the other vertices are of degree 3. Find the number of vertices in G?

5. a) Suppose a graph has vertices of degree 0, 2, 2, 3 and 9. How many edges does the graph have?

b) Give an example of a graph which is Hamiltonian but not Eulerian and vice versa.

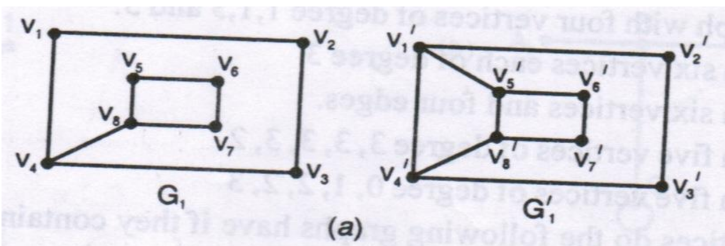
6. a) Let G be a 4 – Regular connected planar graph having 16 edges. Find the number of Regions of G.

b) Draw the graph represented by given Adjacency matrix

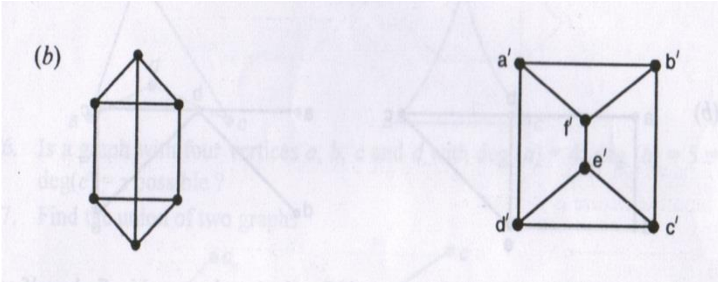
(i) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

7. a) Show that in any graph the number of odd degree vertices is even.

b) Is the following pairs of graphs are isomorphic or not?



8. a) Show that the two graphs shown below are isomorphic?



b) Explain about the rooted tree with an example?

9. a) (i)Find the chromatic polynomial & chromatic number for $K_{3,3}$

(ii) Define Euler circuit, Hamilton cycle, Wheel graph? With examples.

b) Explain any 5 graphs with examples.

10. a) Define regular graph.

b) State handshaking theorem.

c) Define complete bipartite graph.

d) State Euler’s formula.

e) Determine the number of edges in cycle graph C_n with example.